USE OF STATISTICAL THEORY IN PRELIMINARY DESIGN OF SHIPBOARD ELECTRICAL POWER DISTRIBUTION SYSTEMS

MICHAEL E. BISHOP

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by

MICHAEL E. BISHOP, LIEUTENANT, U.S. NAVY

B.S., U.S. Naval Academy

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Signature of Author: ______________________

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Certified by: ______________________

Thesis Supervisor

Accepted by: ______________________

Chairman, Departmental Committee on Graduate Students

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by LT. MICHAEL E. BISHOP, U.S.N.

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ABSTRACT

The thesis is a proposal for a design method which will, when fully evolved, give to the naval power system designer a quantitative means of evaluating relative system merit.

A mathematical model for load analysis is proposed wherein the power requirement of each electrical load is described as a continuous random variable. The sum of these component variables constitutes the total power output of the generating plant.

It is proposed, on the basis of the Central Limit Theorem of Probability, that the density function of the total load requirement may be approximated by the Normal Density function. The mean and variance of this function is then expressed as the sum of the mean and variance of the component variables.

A system study is outlined, based largely on similar work that has been done in the commercial power generation field, which would combine the results of the load analysis with generator reliability data to determine the system reliability of any arbitrary combination of generators. This measure of system reliability is used as the common standard in choosing an optimum system, on the basis of cost and weight, for a particular design.

Numerical examples of some of the proposed calculations are presented. Recommendations for a program of data collection necessary for the full development of the design method and for testing the validity of the theoretical approximations are presented.

Thesis Supervisor: Alexander Kusko

Title: President, Alexander Kusko, Inc.; Lecturer, Department of Electrical Engineering

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I. INTRODUCTION

The rapid rise in commercial and private utilization of electrical power in this country since its first introduction has been paralled by a similar rise in shipboard applications. A logical index of this growth is the measure of installed generator capacity. A ship of the destroyer type in the pre-World War II period might be fitted with a total generating capacity of 300 KW; ship's service generating plants of destroyers built during World War II have recently been up-graded to a 1000 KW capacity; and ships built or building since the war have shown an even greater growth in installed capacity. The reasons for this almost exponential growth are obvious; application of electronics to weapons systems, conversion of machinery auxiliaries to electric drive, increased use of electric power in lighting, air conditioning, galley, and other "hotel loads"; all of these factors have made their contribution to increasing electrification.

It is the effects of this rapid growth which are of prime interest to the ship designer. The obvious effects are the increase in cost, weight, and space of electrical power systems relative to total ship cost, weight, and space. The latest cost estimate used in preliminary design for electric plants in a ship of the DLG class is $13,400 per ton of installed plant. For a nuclear submarine this figure is $9,500 per ton. Of course, the importance of these factors varies with ship type. The space and weight of the electric plant might not be of primary importance in the design of an

aircraft carrier, while in the design of a deep diving nuclear submarine, size and weight of any installed machinery is extremely important. In any case, it is reasonable to state that each of these three factors is of importance in every ship design, and if savings can be made in these areas without sacrifice of other ship characteristics, action taken to effect such savings would be desirable.

To the preliminary designer, there is another effect of this rapid growth in installed capacity which is not as obvious as those already discussed. This is the effect of the growth itself on a particular ship. World War II, with its electronic innovations, accelerated this load growth to such an extent that naval architects found it necessary to install reserve capacity which was a significant percentage of initial requirements to allow for future growth in ship's electrical load. Allowance for load growth in generating plants is not a new concept, of course. Commercial power companies have long recognized the necessity for load growth studies aimed at predicting future requirements. But the problem on shipboard is more complex. First, a naval architect seeks to design his ship as an integral unit, self contained, with all space and weight carefully alloted. In general, it is not possible to set aside space or weight margin for use in future installation of generators and switchboards. Even if this were so, the main power plant itself, which supplies energy to the turbines driving the generators, could not be economically designed with an allowance for such future requirements. Consequently, the

ship as built must be a complete unit, with reserve capacity already installed sufficient to meet anticipated load growth within the useful life of the ship. Design policy in the U.S. Navy has been to increase estimated new ship battle load by a factor of 20% to allow for future load growth. There are, of course, no hard and fast rules to determine the useful life of a ship. Traditionally, this figure for useful life has been around 20 years. Needless to say, an accurate estimate of load growth over a 20 year period is exceedingly difficult to make. It will vary from ship to ship, is dependent on technological innovations not yet introduced, and even on future world political trends. Changes in ship's armament, for example, might not be made in peaceful times, where in times of stress they might be considered all important.

The foregoing has been an attempt to outline for the uninitiated the general problem faced by the designer in preliminary stages of electrical plant design. The method of solution which has been employed by U.S. Navy design agencies consists of three parts: first, estimation of total load requirement based on a study of equipment to be installed, and application of demand factors to the rated load requirements of this equipment; second, application of a 1.2 growth factor to the total load so obtained; and lastly, selection of number and size of generators required from a range of standard sizes, considering largely in qualitative terms such factors as reliability, flexibility, and ease of expansion of the resulting installation.

As is true in most preliminary design, the method relies

heavily on previous experience and is intended to yield a conservative estimate of total load requirements. The method of load analysis yields a set of fixed numbers corresponding to the expected total load for various conditions of operation. These numbers do not provide for the designer a quantitative feeling for the possible range of variation of total load from this value, but they have proven adequate in most cases for designs to date.

Naval engineers have long been aware of the desirability of designing "optimum" systems. In general, it has been impossible to define in quantitative terms the criteria for an optimum system, for a ship is an exceedingly complex engineering structure. The factors to be considered in a design are well known in general terms - cost, weight, reliability, ability to perform desired mission, defensive and offensive capabilities are some of the considerations. Optimization of a ship as a single system still remains largely a qualitative process. But analytical optimization of systems within the ship have been attempted and accomplished with varying degrees of success. One example of a proposed method of optimization of propulsion plants has been codified within the past 5 years and has received some support among designers.[1]

This paper will deal with an approach to the problem of optimization of the electrical plant. The first stage will consist of an exposition of a proposed statistical method of load analysis. It will then be shown how this load estimate may be used in a system study leading to an optimization in

the preliminary design phase of a design.

Numerical examples of some of the calculations will be presented in Appendices A and B.

II. THEORETICAL PROPOSAL

A. Load Analysis

It was stated in the introduction that the present method of preliminary load analysis as employed by the U.S. Navy yields a number representing estimated total kilowatt load requirement for operating conditions of interest. Normally, these conditions are battle, normal steaming, and anchored.

It is the partial intention of this thesis to set forth a proposed method of load analysis which will yield an estimate of mean load under any of the above conditions, as well as a quantitative statistical measure of possible extent of variation from this mean. These measures of mean and variation from mean will be considered as continuous functions of time, so that their behavior during an operating condition transition (such as the change in operational readiness from normal cruising to full battle readiness) may be examined and considered in the design process.

The method will involve the utilization of certain elementary concepts of probability theory. In employing this theory, direct citation of literature will not be made at each point of application. The bibliography contains a list of those recognized texts which were used as reference in this field. During the development, many approximations and simplifications have been made and are pointed out as they occur. The nature of the problem is such that many of these approximations cannot at this time be completely justified,

due to the lack of quantitative statistical data on load behavior.

The total load requirement as seen at any time by the generating plant can be visualized as made up of the summation of the requirements of all the power consuming devices installed in the ship. If a recording wattmeter were placed on the input terminals of one of these devices, a plot of kilowatt power requirement versus time could be obtained for that load. In general, it would be found that this load-time function or signature would be a non-deterministic function, that is, the magnitude of the power requirement at any specified time could not be predicted exactly, as from a fixed analytical expression.

But if we could bring together a large number of identical ships, all operating under the same condition of readiness, and could then record at a prearranged instant the value of input power for this device on each of the ships, this data would yield information that could be used in predicting the behavior of the load.

The theoretical proposal will be based on the concept that these load time signals may be described in fact as stochastic signals, where the value of load requirement at any time t on a particular load is a random variable. We will now define the functions which are necessary to describe the behavior of these random variables so that they may be used in a design process.

1. <u>Probability Density Function</u>

Let us refer to the collection of load-time recordings

which we have obtained in our hypothetical experiment as an ensemble of signals. We may introduce the concept of first probability density function of a random variable as follows:

Set up a range of possible values of load defined as the interval between k_1 and $k_2 + \Delta k_1$ kilowatts. If we then examine the load data at some time on the recordings t_1 and count the number of times, ΔN_1, that the load values have fallen within this range, we can define the first probability density function as

$$p_1(k_1, t_1) \triangleq \lim_{\substack{N \to \infty \\ \Delta k_1 \to 0}} \frac{\frac{\Delta N_1}{N}}{\Delta k_1} \qquad \text{(II-1)}$$

By this definition, the probability density is a function of the value k_1, specifying the sampling range, and the time of observation across the ensemble, t_1. In the case of load requirement of some component load, we would find that this density is a continuous function of the range k_1 over some discrete range of values. Figure I is a sketch of a typical density function indicating the time dependence.

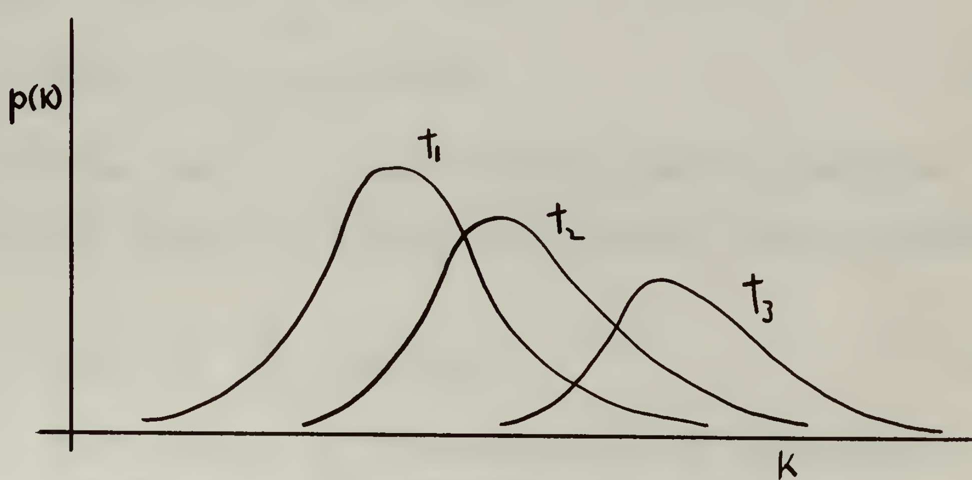

FIGURE I

A Typical Time Varying First Probability Density Function

A knowledge of the first probability density function of a random variable allows us to derive certain other descriptive measures of the behavior of that particular variable. Two of these, which will be used throughout this paper, are the mean, (sometimes called the expected value) and the variance. The expression for these terms will be derived using the same collection of hypothetical data that we considered in this section, the values of load, k, of each recording at some time of observation t_1.

2. Derivation of Mean

If we designate each of the bits of data by k_{1n} where 1 specifies the time of sampling, t_1, and n specifies the member of the ensemble, the ensemble average, mean, or expected value is defined by the equation

$$E[k_1] \triangleq \lim_{N\to\infty} \frac{k_{11} + k_{12} + k_{13} + \cdots\cdots + k_{1N}}{N} \qquad \text{(II-2)}$$

For a random variable with a continuous density function, this expression is equivalent to

$$E[k_1] \triangleq \int_{-\infty}^{\infty} k_{1p}(k_1)dk_1 \qquad \text{(II-3)}$$

For the case of a random variable with a discrete probability density function, the expected value is defined as

$$E[k_1] \triangleq \sum_{i=+\infty}^{\infty} k_{1i}P(k_{1i}) \qquad \text{(II-4)}$$

where $P(k_{1i})$ is the probability of occurrence of the value of load corresponding to k_{1i}

3. Derivation of Variance

The final statistical measure with which we will be concerned is the variance, which is a measure of concentration of data points about the mean.

For simplicity in notation, designate the expected value, or mean, by the symbol m.

Then variance is defined simply as the expectation of the square of the difference between the mean and data points, or

$$\sigma_{k_1}^2 = \int_{-\infty}^{\infty} (k_1 - m)^2 p(k_1) dk_1 \qquad \text{(II-5)}$$

We will also use another expression, the standard deviation, where this is defined as the square root of the variance.

Standard Deviation $= \sigma_{k_1}$

4. Shipboard Considerations

The three describing functions, probability density function, expected value and variance, which have been defined are sufficient to describe or predict the behavior of the load requirement at any point in the system, whether it be a component input, distribution feeder, or main generator bus tie. In the most general case, as has been indicated in the definitions, a stochastic process is assumed to have time dependent statistics. Within the framework of our ship design, we will postulate a hypothetical range of time extending, for example, from an anchored condition through normal cruising conditions to full battle condition. Over this range of time we would expect that many of the component

load requirements, defined as stochastic signals, would have time varying values of mean and variance. However, if we consider the range of time from cruising through battle conditions, we suspect that there are a number of shipboard loads that would not be affected appreciably by such a transition.

Thus if we examine a component load such as a missile launcher drive motor, the statistical behavior could be expected to change markedly on a transition from cruising to battle. On the other hand, certain machinery loads, such as feed booster pumps, lube oil pumps, even steering gear pump drives should be relatively unaffected by such a transition, and hence an assumption of time independence for these loads over our hypothetical transition period might be justified. This leads in general to a considerable simplification in the estimation process.

Obviously, the time period from cruising through battle might not be the condition of greatest interest from the standpoint of generator sizing in all ship types. The proposed analysis method is not affected by such a difference, requiring only that the correct period for the ship is chosen, a decision which is obvious in most cases, but certainly amenable to experimental verification if a similar ship already exists.

B. Derivation of Total Load Description

The second problem to consider in applying the method to a design is that of deriving a total load description from the component load descriptions.

Given a random variable made up of the sum of N

independent random variables, the mean of the sum is equal to the sum of the means and the variance of the sum is equal to the sum of the variances.[2] That is, if

$$Y = x_1 + x_2 + x_3 \; . \; . \; . \; . \; . \; x_N \qquad \text{(II-6)}$$

where the random variables x are statistically independent.

Then Y will have a mean given by

$$m_Y = \sum_{i=1}^{N} m_{x_i} \qquad \text{(II-7)}$$

and a variance

$$\sigma_Y^2 = \sum_{i=1}^{N} \sigma_{x_i}^2 \qquad \text{(II-8)}$$

Two random variables x and y are said to be statistically independent if the joint probability density function of x and y can be expressed as the product of the individual probability density functions of x and y, or $p(x,y)=p(x)p(y)$.

In a physical sense, for our load analysis problem, an assumption of statistical independence of the various component loads implies that the value of load requirement of any one of the components is not affected by the load requirements of the remaining components. This may in fact be a gross approximation. We sense that the load requirements of components within some system might be intimately related as the requirements on that system vary. There are techniques available for carrying out the operations of Eq. 6, 7, and 8 for sums of random variables which do not exhibit statistical independence. However, they involve a considerable degree of mathematical sophistication and, perhaps of more importance, an increase in the amount of basic knowledge of the statistical

behavior of the individual component loads. This proposal is an attempt to approach a statistical estimation of total load. Before complicating the analysis, it might be desirable to determine in an actual design the effect of the simplifying assumptions on the results.

Assuming statistical independence, we may now combine the results of the component descriptions to obtain the mean and variance of the total load requirement. To employ these results in a design process, it will be necessary to know the first probability density function of the total load also. This will allow the prediction, in terms of a probability, that the total load will exceed some design value, since

$$\text{Prob. } (L \geqslant L_r) = \int_{-\infty}^{L_r} p(L)\, dL \qquad \text{(II-9)}$$

where L = Total Load Requirement

$p(L)$ = First probability density function of L

L_r = Design or Reference value

Determination of the density function of a random variable which is a linear sum of random variables is a classical problem in statistical theory. One method involves the use of transforms sometimes called characteristic functions.[2] In essence, the method involves taking the Fourier Transform of each of the component density functions. If these components are assumed to be statistically independent, then the Fourier Transform of the random variable representing the sum is equal to the product of these component transforms. Having so obtained the Fourier Transform for the sum, this function may be inverse transformed to yield

the first probability density function of the sum. Here, again, the method requires a complete knowledge of the first probability density functions of the components. Furthermore, it requires that they be Fourier transformable. Some experience in application of the method might prove that such a precise method is necessary to achieve valid results. This thesis proposes that a second major approximation in the method of analysis be made at this point, through application of the Central Limit Theorem.

A random variable is said to have a normal distribution if it has a first probability density function of the form[3]

$$p(k) = \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(k-m)^2}{2\sigma_k^2}} \qquad \text{(II-10)}$$

where as we have previously mentioned

σ_k = standard deviation

m = mean

In many practical applications of statistical theory, it has been found that the density function of a large number of observations made upon the outcome of a repetitive experiment tends toward the normal distribution as the number of observations is increased. To put this qualitative statement in terms of a more definitive application to our problem, consider the application of the Central Limit Theorem to the following example:

If we have $k_1, k_2, \ldots k_i \, .. k_N$ independent random variables, each of which has a mean m_i and standard deviation σ_i, the

sum

$$L = \sum_{i=1}^{N} k_i \tag{II-11}$$

will, as in II-6 and II-7 have a mean and variance

$$m = \sum_{i=1}^{N} m_i \tag{II-12}$$

$$\sigma^2 = \sum_{i=1}^{N} \sigma_i^2 \tag{II-13}$$

By the Central Limit Theorem[2,4,5], as the number of variables N approaches infinity, the sum L will have a normal probability density function

$$p(L) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(L-m)^2}{2\sigma^2}} \qquad -\infty < L < \infty \tag{II-14}$$

For the case of a typical ship's electrical system, the number of component loads may range from 250 in a DL type to several thousand in a large carrier. These loads range in power requirement from a fraction of a KW to 50-60 KW, with the contribution of the largest single load being of the order of 5%.* The normal approximation has been successfully made in many engineering cases where sample size or N was a finite number. We will make this assumption at this point, without presenting a rigorous mathematical defense. The test of the assumption lies in the collection of sufficient data to enable determination of its validity. It is desirable at this point, since it considerably simplifies the approach to

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*This excludes consideration of extreme cases where a single load may make up a considerably greater percentage of total load. Often in such cases such a load is provided with an integral power supply other than ship's service capacity.

load analysis and provides a total load probability density function that is extremely simple to work with.

At this point we might consider the form of the normal probability distribution itself. It can be seen from Eq. II-10 that the density function is symmetrical about the mean m and has an infinite range of possible load. However, depending on the value of m and standard deviation, an actual plot of the function rapidly approaches zero for increasing values of magnitude of (L-m). Hence, though the density function always assigns a finite probability to values of load anywhere from $-\infty$ to ∞, the probability of very large deviations from m can be small enough to be neglected for practical purposes, as will be shown in the example in Appendix B.

We are now in a position to outline in step-by-step form the proposed method of load analysis. In summary, these steps are as follows:

A. Tabulation of and investigation of component loads.

This will consist of a listing of loads to be installed, with a statistical description of each of these loads based upon a study of existing installations or in the case of a radically new application, a best estimate.

B. Linear addition of the mean and variance of each of these loads to obtain, with the assumption of a normal distribution, the statistical description of total load behavior. This calculation would be carried out across the time range of interest, as discussed, to determine the effect of operating conditions on load behavior.

III. INVESTIGATION OF INDIVIDUAL LOAD STATISTICS

As previously stated, it will be necessary to determine experimentally or postulate for each component load or load group (as in the case of lighting loads) a statistical description consisting of the first probability density function, the expected value or mean, and the variance of the load about the mean.

The example used to illustrate the proposed load analysis indicated a possible experimental method (actually hypothetical) for determining these statistics. Examination of the "method" soon reveals that it would be impossible to apply in an actual case, since it requires the existence of an ensemble consisting of a large number of ships. Furthermore, it requires the existence of ships which are actually still in the preliminary design phase, since as we know we are concerning ourselves with the design of the electrical systems.

It is obvious then that assumptions must be made in the preliminary design phase for the required statistical descriptions. This is not to say that these assumptions cannot be heavily based upon experimental evidence. As this paper will attempt to point out, a large number of shipboard loads will exhibit essentially the same statistical behavior regardless of the ship in which they are installed. Lighting loads, air conditioning loads, boat winches, fire pumps, auxiliary electrical equipment in machinery spaces, even search radars and radios could be expected to have, aside from magnitude and time scales, the same form for their

statistical description on most surface warships. Investigation of the design characteristics of the equipment to be installed, if available, together with investigation of data available on similar equipment already in use would then be used to determine the necessary information as regards magnitude and time.

The designer is not apt to encounter a component load which is entirely different from anything that has been previously installed. If he does so, he is in the same position with this proposed method of load analysis as in the conventional technique, since he must use such data on the new equipment as is available to determine the description he requires, whether it be a load factor or a first probability density function of load and time.

A. Classification and Description of Loads

In order to establish a set of characteristic statistical descriptions to be used in application of the method, we will first consider the general types of electrical loads which are present in a ship's electrical system. The first classification is on the basis of physical configuration, as follows:

1. Motor Loads
    a. Inductive
    b. Synchronous
2. Resistive Loads
3. Electronic Loads

The second classification will be on the basis of the effect of an operating condition transition on the load statistics, where the term operating condition transition is

used to describe a change in the ship's battle readiness, bringing about a major change in the electrical load structure. In a surface warship, a typical transition of interest is the change from normal steaming conditions to full battle readiness; in a submarine, a somewhat analogous transition would be the change from slow speed passive search operation to a high speed attack mode. Under this classification, loads will be designated "no immediate effect" or "immediate effect."

If we were considering a DL-type surface warship, then we would classify the gun mount drive motors as "induction motor-immediate effect"; sonar loads as "electronic-no immediate effect"; lighting loads as "resistive-no immediate effect"; and fire pumps might be "induction motor-no immediate effect."

Let us now investigate each of these sub-classifications, the objective being to derive a general statistical description for each in both the steady state and transient operating conditions. In general, the following derivations will apply most closely to the surface warship electrical systems - the general nature of the submarine problem seems to be one that might be approached simply through the application of the method to a steady state investigation.

B. Inductive Motor Loads

We are interested in the following characteristics of such a load:

1. Number of times that the motor might be energized during the period of interest.

2. Length of time that the motor might be expected to stay energized.

3. Transient behavior of the motor (or motor-starter combination) on energization.

4. Possible range of motor load during operation.

5. Manner in which load requirement changes during operating transient.

We will first examine two hypothetical cases and then see how for a particular motor, study of items 1 through 5 may help us apply the results to an actual case.

CASE I:

Consider a load which has the classification "induction motor - immediate effect." Let us assume that this motor has a starting characteristic which can be described by Figure II below, where k(t) is the time-load characteristic.

$$k(t) = Nu_{-1}(t) - (N - n)u_{-1}(t - T) \qquad \text{(III-1)}$$

$u_{-1}(t)$ is a unit step occuring at t = 0

N is transient starting peak power requirement in KW

n is steady load requirement in KW

T is duration of starting transient in seconds

k(t) is load requirement at any time t in KW

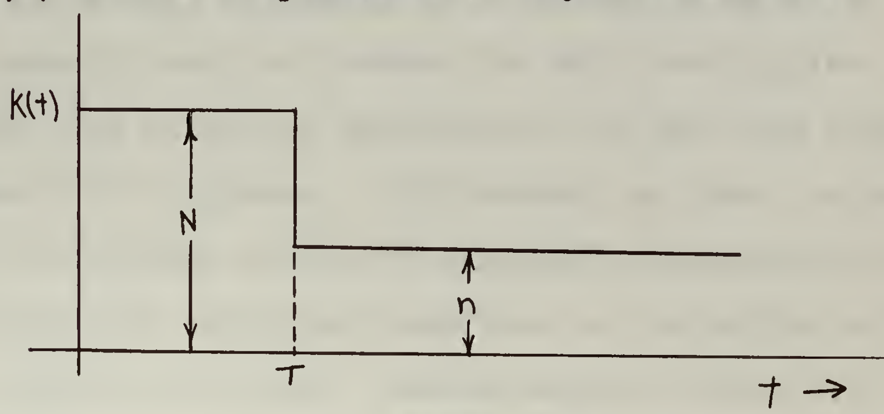

FIGURE II

Starting Transient Model, Induction Motor

Now let us superimpose this transient description on the operating condition transition time scale as in Figure III below, where k(t) is now

$$k(t) = Nu_{-1}(t-A) - (N-n)u_{-1}(t-T-A) \qquad \text{(III-2)}$$

and A is the time of energization of the motor, and t = 0 corresponds, for example, to the time of sounding "General Quarters", that is, the initiation of the operating condition transition.

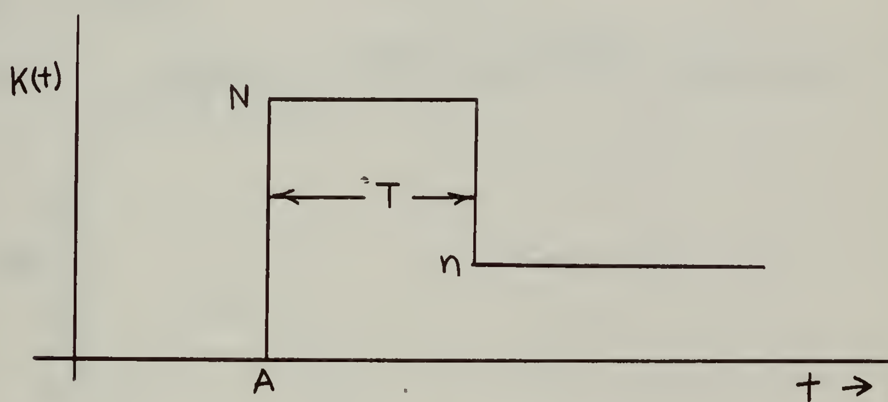

FIGURE III

Shifted Induction Motor Model

Essential to the further development of the transient case is the postulation of a mathematical description for A, the time of energization of the various equipments. If we consider the actual situation in a surface warship, we can deduce certain practical bounds for this description. First, we know from practical experience that the time A has a range of from 0 to 3 minutes. Furthermore, we know, or suspect, that for stations which are originally unmanned the load is more apt to be energized somewhere in the middle of this range than at the ends. Lacking specific data, we will use this intuitive experience to define energization time, A, as

a random variable with a Gamma distribution, leading to a first probability density function as follows:[3]

$$p(A) = \frac{1}{\beta^{\alpha+1}\Gamma(\alpha+1)} A^{\alpha} e^{-A/\beta} \qquad \begin{pmatrix} \alpha > -1; \ \beta > 0 \\ 0 \leqslant A < \infty \end{pmatrix} \quad \text{(III-3)}$$

where α and β are arbitrary constants which determine the shape of the density curve. If we choose $\alpha = 1$, $\beta = 0.5$, the resulting probability density curve has the shape shown in Figure IV. Substituting this value of β and α in Eq. III-3, we have for p(A):

$$p(A) = \frac{1}{(.5)^2\Gamma(2)} Ae^{-2A} = 4Ae^{-2A} \qquad (0 \leqslant A < \infty) \quad \text{(III-4)}$$

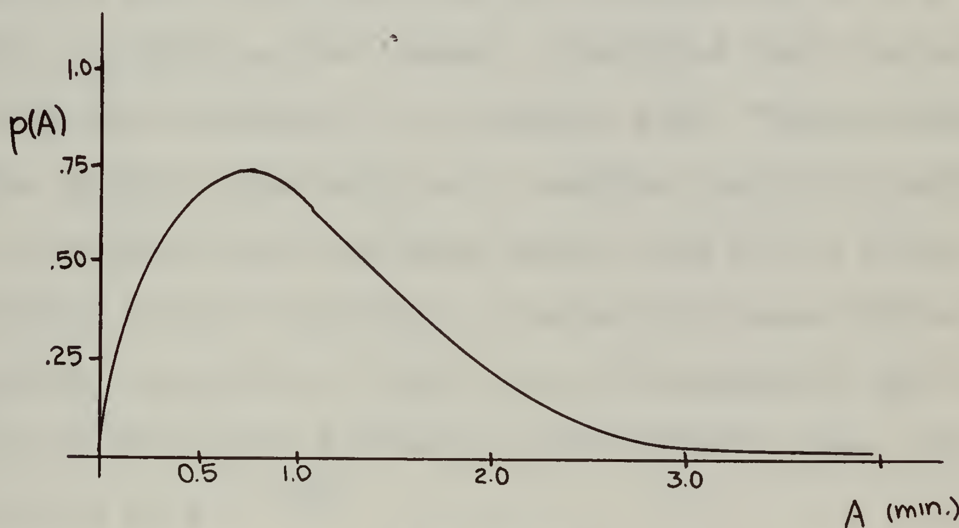

FIGURE IV

Gamma Density Function of Specified Form

If we consider this density function with the physical situation in mind, we see that A will lie essentially between 0 and 3 minutes, and is most likely to occur in the vicinity of 0.5 to 1 minute. Remembering that

$$\text{Prob}(A \leqslant T) = \int_0^T p(A)dA, \ x \qquad \text{(III-5)}$$

we can plot the distribution function, $P(A) = \text{Prob}(A \leqslant T)$, as in Figure V.

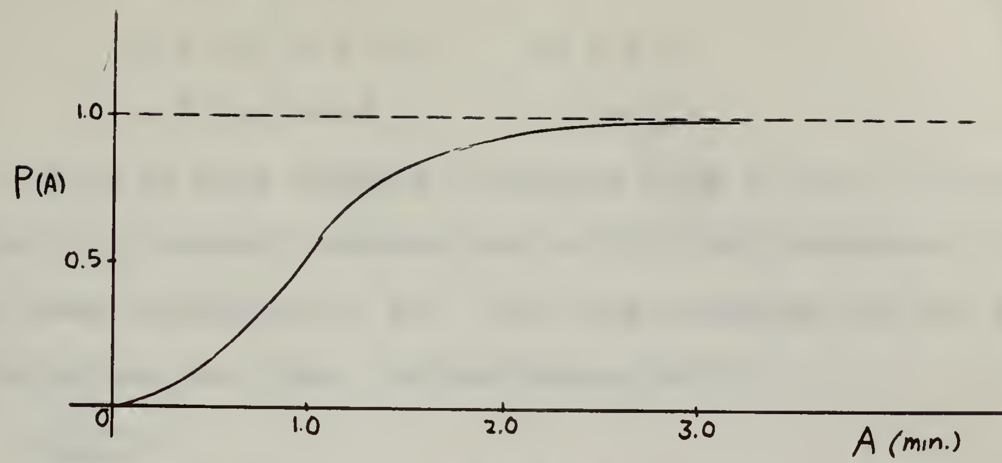

FIGURE V

Gamma Distribution Function

We see from this plot that the probability of $A \leq 3$ min. is 0.983, indicating the extreme likelihood that the motor will have been energized by 3 minutes after "General Quarters."

The problem remaining is to combine the above information to determine for our ideal motor load k(t) a first probability density function, or actually a mean and variance at discrete intervals of time t on our transition time scale.

If we return for a moment to the general case, let

$$p(A) = C_1 A e^{-C_2 A} \tag{III-6}$$

Then the expected value of load at any value of time T_1 on our transition time scale can be defined as:

$$E[k(T_1)] = \int_0^{\infty} p(A)\, k(A)\, dA \tag{III-7}$$

where k(A) here means the value of load, k, at time T_1, given that the transient started at time A. By inspection of Figure III we see that for any given value of time T_1, k takes on discrete values as a function of transient time, T, and energization time, A, or

$$k(A) = 0 \text{ for } A > t \qquad \text{(III-8)}$$

$$= n \text{ for } t-A > T \qquad \text{or } t-T > A$$

$$= N \text{ for } t-A \leq T \qquad t-T \leq A$$

Here we have changed variables from T_1 to t, in order to obtain our general expressions in E(k) and Variance. Employing these relations in Eq. III-7 and carrying out the required integration, we find, for two ranges of t:

<u>$t < T$:</u>

$$E(k) = \frac{N\,C_1}{C_2^2}\,[1 - e^{-C_2 t}(C_2 t + 1)] \qquad \text{(III-9)}$$

<u>$t \geq T$:</u>

$$E(k) = \frac{nC_1}{C_2^2}\left\{1 - e^{-C_2(t-T)}[C_2(t-T) + 1]\right\} + \qquad \text{(III-10)}$$

$$\frac{NC_1}{C_2^2}\left\{e^{-C_2 t}(-C_2 t-1)+e^{-C_2(t-T)}[1+C_2(t-T)]\right\}$$

We can derive the expressions for variance in the same ranges of t from the definition of variance as

$$\sigma_k^2 = E\left\{[k - E(k)]^2\right\} \qquad \text{(III-11)}$$

which reduces to

$$\sigma_k^2 = E(k^2) - [E(k)]^2 \qquad \text{(III-12)}$$

where

$$E(k^2) = \int_{-\infty}^{\infty} p(A)k^2(A)\,dA \qquad \text{(III-13)}$$

Carrying out the operation of Eq. III-13 we find for our two ranges of t:

<u>$t \leq T$:</u>

$$E(k^2) = \frac{N^2 C_1}{C_2^2}\,[1 - e^{-C_2 t}(C_2 t + 1)] \qquad \text{(III-14)}$$

[illegible]

$\underline{t \geq T:}$

$$E(k^2) = \frac{n^2 C_1}{C_2^2}\left\{1-e^{-C_2(t-T)}[C_2(t-T) + 1]\right\} +$$

$$\frac{N^2 C_1}{C_2^2}\left\{e^{-C_2 t}(-C_2 t-1) + e^{-C_2(t-T)}[1+C_2(t-T)]\right\} \quad \text{(III-15)}$$

Now carrying out Eq. III-12 on our results for E(k) and $E(k^2)$, we find for the variance

$\underline{t \leq T:}$

$$\sigma_k^2 = \frac{N^2 C_1}{C_2^2}[1-e^{-C_2 t}(C_2 t+1)] - \frac{N^2 C_1^2}{C_2^4}[1-e^{-C_2 t}(C_2 t+1)]^2 \quad \text{(III-16)}$$

$\underline{t \geq T:}$

$$\sigma_k^2 = \frac{n^2 C_1}{C_2^2}\left\{1-e^{-C_2(t-T)}\left[C_2(t-T)+1\right]\right\} + \frac{N^2 C_1}{C_2^2}\left\{e^{-C_2 t}(-C_2 t-1) + e^{-C_2(t-T)}\left[1+C_2(t-T)\right]\right\}$$
$$-\left[\frac{n C_1}{C_2^2}\left\{1-e^{-C_2(t-T)}[C_2(t-T)+1]\right\} + \frac{N C_1}{C_2^2}\left\{e^{-C_2 t}(-C_2 t-1) + e^{-C_2(t-T)}[1+C_2(t-T)]\right\}\right]^2$$

(III-17)

If we remember from our definition of p(A) in Eq. III-6 that we have set the value of α in 3-3 to be equal to 1, it is readily seen that the ratio of C_1/C_2^2 is constrained to be one also. Equations III-9, III-10, III-16 and III-17 are then simplified somewhat. Furthermore, if we normalize the mean and variance with respect to steady state load, n, we obtain the following relations:

$\underline{t \leq T:}$

$$E(k)/n = (N/n)[1 - e^{-C_2 t}(C_2 t + 1)] \quad \text{(III-18)}$$

$$\sigma_k^2/n^2 = (N/n)^2[e^{-C_2 t}(C_2 t+1)][1-e^{-C_2 t}(C_2 t+1)] \quad \text{(III-19)}$$

[illegible]

$$[illegible]$$

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t $\geq$ T:

$$e(k)/n = \left\{1 - e^{-c_2\tau}(c_2\tau + 1)\right\} + (N/n)e^{-c_2 t}\left\{e^{-c_2 T}[1 + c_2\tau] - (c_2 t + 1)\right\} \quad \text{(III-20)}$$

$$\begin{aligned}\sigma_k^2/n^2 = {} & e^{-c_2\tau}(c_2\tau+1)[1-e^{-c_2\tau}(1+c_2\tau)] \\ & + (N/n)^2[e^{-c_2\tau}(1+c_2\tau) - e^{-c_2 t}(1+c_2 t)] \\ & [1 + e^{-c_2 t}(c_2 t+1) - e^{-c_2\tau}(c_2\tau+1)] - \\ & - 2(N/n)[1-e^{-c_2\tau}(c_2\tau+1)][e^{-c_2\tau}(1+c_2\tau) - e^{-c_2 t}(c_2 t+1)] \end{aligned} \quad \text{(III-21)}$$

where τ = t-T

In Equations III-18 through III-21 we have the necessary expressions for the statistical description of a load considered in Case I, that is "induction motor-immediate effect." Application of this ideal case to a physical motor load will be considered in Appendix A. Figure VI is a plot of normalized load mean and variance versus time for the following assumed physical parameters:

C_1 = 4.0 T = 0.2 minutes

C_2 = 2.0 N/n = 3.0

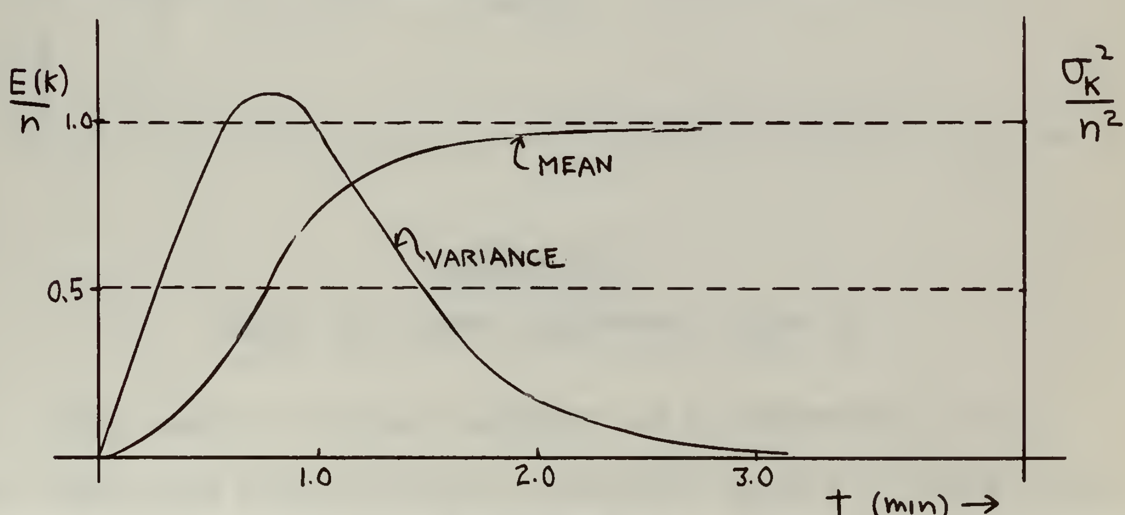

FIGURE VI

Typical Curve of Normalized Mean and Variance vs. Time for Case I

We will now consider our second case under Induction Motors, or "Induction Motor-no immediate effect."

CASE II:

Here we will attempt to construct a mathematical model for the induction motor which is not immediately affected by an operating condition transition, or more generally, the case of any induction motor load during steady state operation. Again we should stress that the term "steady state" in this context refers to the absence of transients in ship operating conditions, and not to the existence of a steady level of power requirement at the load.

For our model of motors which might fit the classification of Case II, we will use a random process that would have as a member of its ensemble the signal sketched in Figure VII below.

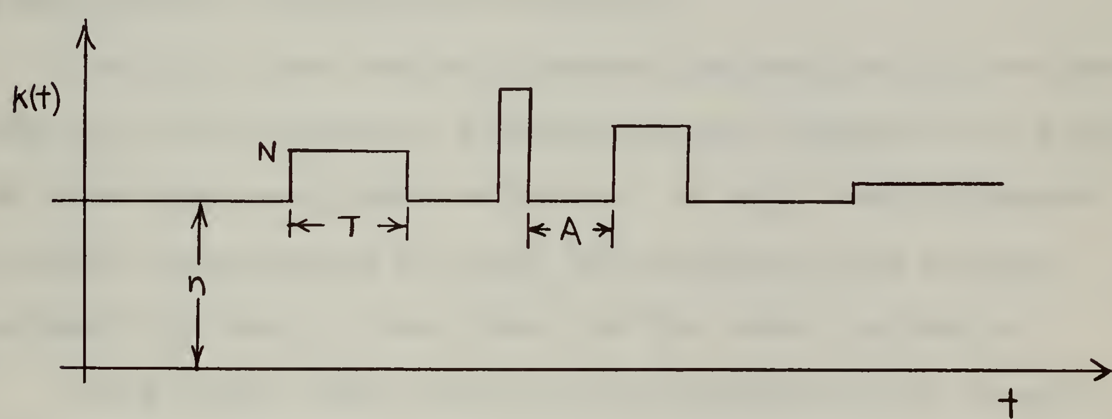

FIGURE VII

Model for Load Signature, Case II

Note that the signal consists of a reference level n upon which are superimposed rectangular pulses of height N-n, length T, and time between occurence A. We will characterize this signal by three statistically independent random

variables, namely A, T, and N/n. The value of n may range between 0 and some finite upper limit, but is fixed for any given motor. Obviously, the range and magnitudes of the variables are strongly dependent on the use to which the motor is being put; however, we will make the following simplifying assumptions to aid in understanding and applying the mathematical model:

a. The basic form of the probability density function describing the variables A and T will be the same from motor to motor.

b. Differences between motors will then be expressed in terms of range of A and T, height of n, and probability density and range of N/n.

c. As stated, A, T, and N/n are considered to be statistically independent variables.

We will first derive a general expression for the mean and for the variance of a random process defined as a function of more than one random variable. We will then determine specific expressions for mean and variance with assumed probability density functions for the three variables.

This model lends itself to an analysis which is particularly simple in concept and in application. Without concerning ourselves with the ultimate form of the probability density function of k(t), we can say from the definition of expectation that

$$E(k) = P_a\ k(A) + P_t\ k(T) \qquad \text{(III-22)}$$

where

P_a = probability that at any time t, we have no pulse

P_t = probability that at any time t there is a pulse

k(A) = value of load given that there is no pulse

k(T) = value of load given that there is a pulse

The first term becomes, in our model, simply

$$P_a k(A) = nP_a \tag{III-23}$$

The second term is somewhat more complex, since we have assumed that k(T) can take on a range of values of N. It can be shown that

$$P_t k(T) = P_t \int_n^\infty N p(N) dN \text{ for a continuous distribution in N}$$

or

$$P_t k(T) = P_t \sum_k N_k P(N_k) \text{ for a discrete distribution in N}$$

which of course reduces to

$$P_t k(T) = P_t \; E(N) \tag{III-24}$$

Consider now the meaning of the terms P_a and P_t. We see that for our simple model these probabilities can be expressed in analytical form as

$$P_a = \lim_{\Delta t \to \infty} \frac{\text{Total time in } \Delta t \text{ with no pulse}}{\Delta t} \tag{III-25}$$

$$P_t = \lim_{\Delta t \to \infty} \frac{\text{Total time in } \Delta t \text{ with a pulse}}{\Delta t} \tag{III-26}$$

In a similar fashion, we can say for $E(k^2)$:

$$E(k^2) = P_a k^2(A) + P_t k^2(T) \tag{III-27}$$

or

$$E(k^2) = P_a \, n^2 + P_t \int_n^\infty N^2 p(N) dN \text{ for a continuous variate N} \tag{III-28}$$

$$= P_a \, n^2 + P_t \sum_k N_k^2 P(N_k) \text{ for a discrete variate N}$$

Then for variance, we have from Eq. III-12:

$$\sigma^2 = P_a n^2 + P_t k^2(T) - [P_a\ n + P_t k(T)]^2 \qquad \text{(III-29)}$$

$$= P_t k^2(T) - 2P_a P_t nk(T) - [P_t k(T)]^2$$

We have then in applying this model the problem of determining P_a, P_t, and the distribution of N. Equations III-25 and III-26 express the mathematical definition of P_t and P_a. We can readily visualize the type of data necessary to provide us with an estimate of these two probabilities. Similarly, we see from Equations III-24 and III-28 that we are interested in the expected value of N and N^2 in evaluating E(k) and variance. If we can find this expected value (mean) from observed data on previous installations, or from an assumed statistical description of N, we are in a position to evaluate mean and variance for loads fitting Case II of our induction motor model.

We will find it to our advantage in applying this model to an actual load analysis to express N and n as fractions of name plate rating in KW. Then we will be able to set up normalized design tabulations which have a more general utility.

This completes the study and postulation of our models for analysis of induction motor loads. We cannot be sure, without extensive statistical observations, whether or not these two simple cases are sufficient to adequately describe all of the motor loads that the designer may encounter. However, in the absence of such data, these models, based largely on an intuitive feeling for the physical situation,

at least may point the way to the proper form.

C. Resistive Loads:

We will define for the purposes of this paper "resistive" loads to be those which are characterized by essentially instantaneous reaction to demand changes, and react with a step function in load requirement to these changes. As in the case of induction motor loads we will consider two cases in developing the necessary mathematical models. In a large part the derivations to follow are quite similar to those which were discussed in some detail previously. Consequently, details of the procedure are omitted where they would be only repititious.

CASE I:

Here we will construct a model of a load which might be classified "resistive-immediate effect." Figure VIII corresponds to a typical member of the ensemble of time functions for such a load, during an operating condition transition commencing at time t = 0. Note that the trace is quite similar to the model for induction motor, Case I, except that the load increase occurs without a transient starting overshoot.

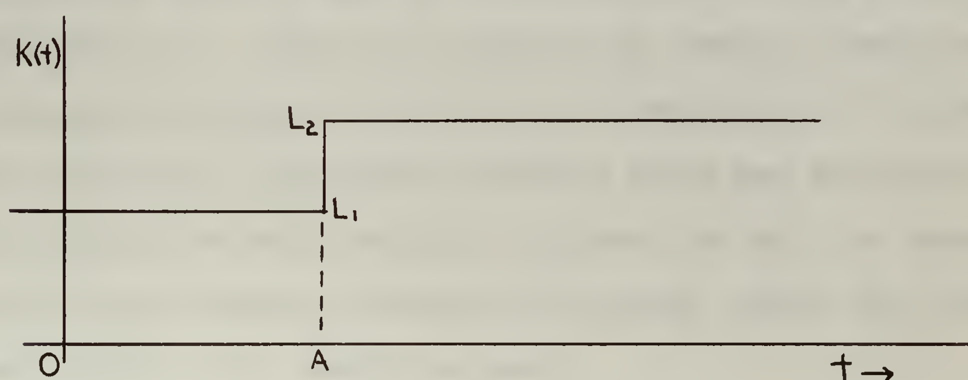

FIGURE VIII

Model for Resistive Load, Case I.

If we define the time signal k(t) to be a function of three random variables, L_1, L_2, and A, we can write immediately for the mean and variance of k(t):

$$E[k(t)] = \int_{t}^{\infty} p(A)dA \int_{L_1} L_1 p(L_1)dL_1 + \int_{0}^{T} p(A)da \int_{L_2} L_2 p(L_2)dL_2 \qquad \text{(III-30)}$$

where p(A) = first probability density function of A.

$p(L_1)$ = first probability density function of L_1.

$p(L_2)$ = first probability density function of L_2

$\int_{L_1}$ = symbol for integral over the range of L_1, etc.

and for $E[k^2(t)]$:

$$E[k^2(t)] = \int_{t}^{\infty} p(A)dA \int_{L_1} L_1^2 \, p(L_1)dL_1 + \int_{0}^{t} p(A)dA \int_{L_2} L_2^2 p(L_2)dL_2 \qquad \text{(III-31)}$$

We will introduce a new probability density function for the random variable whose value is turn-on time, A. This will be the exponential density function, which has the form

$$p(A) = Ce^{-CA} \qquad \text{(III-32)}$$

where C is some arbitrary constant, chosen in our case to match as nearly as possible the physical situation we wish to describe. We will also use the density function for A developed previously and expressed as Eq. III-6:

$$p(A) = C_1 \, A \, e^{-C_2 A} \qquad \text{(III-6)}$$

remembering that C_1 and C_2 are dependent by the relationship $C_1/C_2^2 = 1$. The use of these two density functions for A is intended to provide sufficient flexibility to describe loads which are controlled directly from the distribution switchboard, or other manned stations, as well as those controlled from normally unmanned stations, where the function given in Eq. III-6 would be used.

Considering Eq. III-32 in more detail, if we assume for

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manned stations that the change in load upon an operating condition transition occurs such that A has an effective range of approximately one minute, we see that a choice of C = 2 provides a density function that will conform in range at least to an approximate picture of this physical situation.

Figure IX is a plot of p(A) with a C of 2.0. Since the probability that A less than or equal to 1.0 is the area under the density curve from 0 to 1.0, we see that this probability is 0.865.

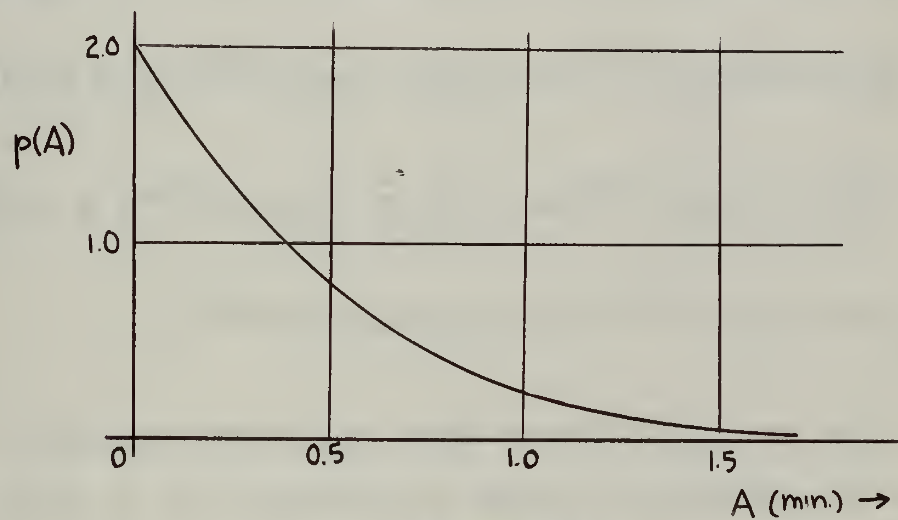

FIGURE IX

Exponential Density Function

By simple mathematics, we can show that

$$\int_0^t p(A)dA = 1 - e^{-Ct} \text{ for } p(A) \text{ as in Eq. III-32}$$
$$= 1 - e^{-C_2 t}(1 + C_2 t) \text{ for } p(A) \text{ in Eq. III-6}$$

$$\int_t^\infty p(A)dA = e^{-Ct} \text{ for Eq. III-32}$$
$$= e^{-C_2 t}(1 + C_2 t) \text{ for Eq. III-6}$$

Using these results, we have for the case where A has an exponential probability density function:

$$E[k(t)] = e^{-Ct}\, E[L_1] + [1 - e^{-Ct}]\, E[L_2] \qquad \text{(III-33)}$$

and

$$E[k^2(t)] = e^{-Ct}\ E[L_1^2] + [1 - e^{-Ct}]\ E[L_2^2]$$

so that

$$\sigma_k^2(t) = e^{-Ct}E[L_1^2] + [1-e^{-Ct}]E[L_2^2] -$$

$$\left\{e^{-Ct}\ E[L_1] + 1 - e^{-Ct}E[L_2]\right\}^2 \qquad \text{(III-34)}$$

And for the case where A has a Gamma distribution:

$$E[k(t)] = e^{-C_2t}(1 + C_2t)E[L_1]+[1-e^{-C_2t}(1+C_2t)]E(L_2) \qquad \text{(III-35)}$$

$$E[k^2(t)] = e^{-C_2t}(1+C_2t)\ E[L_1^2]+[1-e^{-C_2t}(1+C_2t)]E(L_2^2)$$

and

$$\sigma_k^2(t) = e^{-C_2t}(1+C_2t)\ E[L_1^2]+[1-e^{-C_2t}(1+C_2t)]E(L_2^2) \qquad \text{(III-36)}$$

$$- \left\{e^{-C_2t}(1+C_2t)E(L_1)+[1-e^{-C_2t}(1+C_2t)]E(L_2)\right\}^2$$

This completes the model for Case I. We will consider the form of the probability density functions for L_1 and L_2 when we apply the model to an actual case. Note that we have not restricted the density function of these load values as far as their form is concerned; that is, they may be continuous or discrete random variates, or even constants. We have made the implicit assumption, as has been made repeatedly in the foregoing development, that they are statistically independent random variables, as well as being independent of the random variable A.

<u>CASE II</u>:

Experience and study of the various loads that might fit the classification "resistive - no immediate effect" indi-

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cates that the model already developed for Case II of induction motors might be applicable in these cases also. Some loads which we recognize immediately as being in this group are certain lighting loads, galley range loads, and heaters. The model for Case II induction motors appears to have sufficient generality to fit these loads. This, of course, as is the case with all the assumptions made before this point, is not rigorously defensible with data which is presently available.

D. Electronic Loads

There are a number of general characteristics of electronic loads which should be considered before attempting to derive descriptive models to be used in determining their contribution to total load. The first of these is the question of load power supply. A rather large percentage of the total load requirement of fire control electronic equipment in modern warships is supplied by 400 cycle motor generator sets. Where this is the case, we should be able to examine the load requirement from the point of view of the power required by the driving motor of the M-G set. Of course, the power requirement of the motor is a direct function of the needs of the equipment which it serves. The remainder of electronic loads (those not served by M-G sets) receive their power from the ship's service distribution system through transformers. The second point to be made is that, in general, we will find that electronic loads, with the possible exception of fire control equipment, tend to be deterministic in nature. In the case of a search radar, for

example, we know within fairly precise bounds its power requirement as a function of time. Similarly for sonars, if we postulate a search mode for their operation, which is a reasonable thing to do. The third and final point to be made is that, of the electronic loads, the only ones which we would expect to be immediately affected by an operating condition transition are the fire control radars and computers.

We will state at this point then, that based upon the above general comments, we will describe the electronic load in one of the following ways:

a. Through its effect on an induction motor load

b. As a resistive type load, if supplied from the line

c. As a deterministic load requirement

IV. SYSTEM STUDY

This chapter will be concerned with the use of the load analysis results in an overall system study. This study will point the way to an optimum selection of generating unit capacity on the basis of weight and space, cost, and the all important but often nebulous criteria of system reliability.

The past 25 years have seen a growth of interest in the application of probability methods to the problems of fixed station power distribution. Reference[6] presents a comprehensive survey and bibliography of the work which has been done in this field to date. With the advent of the digital computer as an aid in the solution of engineering problems, the methods have seen increased acceptance in the actual solution of practical problems such as calculation of reserve allowance, effect of system inter-connections, and effect of capacity and outage rates of individual units on system reliability.

Many of the concepts which have been developed for use in fixed station power systems can be applied to the shipboard system. As every marine electrical engineer knows, however, the problems involved in a ship's electrical system are in many respects quite different from those encountered in large systems ashore. These distinctions must be kept firmly in mind when applying shore station solutions to shipboard problems. Some of the obvious differences are listed below:

a. Shipboard generating plants are faced with very wide variations in power demand, these variations occurring

in relatively short intervals of time.

b. The generator's energy source in shore stations is commonly combined with the generator so as to form an integral unit. In most marine applications, the generator receives its energy from the main propulsion plant. This has an effect on component reliability.

c. The effects of possible battle damage and the continuing effects of a marine environment must be considered by the ship designer. These problems do not enter in a fixed station design.

d. Shore power capacity is normally upgraded by the addition of generating units. While the pace of modern technology dictates that he must allow for load growth over the life of the ship, present practice does not allow the marine designer the freedom to add at some future time additional generating units to the ship.

e. Loss of load to a utility company may mean customer ill-will and resultant financial difficulty. On a warship, loss of load can well be a disaster.

If a valid design procedure is to be developed for shipboard systems, either as an adaptation of existing theory or an original deviation from that theory, it must reflect the effects of the above, as well as other shipboard constraints.

A. Development of System Model

We will adopt for our definition of system reliability the one which is commonly credited to Calabrese[7]. He expresses system reliability as a function of the probability of loss of load to all or some part of the system. For this

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paper, then, we define reliability, $R(t)$, as:

$$R(t) = 1 - P_D(t) \qquad \text{(IV-1)}$$

where

$P_D(t)$ = the probability that at time t the total load demand exceeds the available generating capacity, the condition which we will consider to result in loss of load to the ship

For our first model, we will assume that the generating capacity is made up of the contributions of N generators of equal capacity. The effect of emergency or spare generators will be considered in a second model. We can now describe the physical status of the generating plant by N + 1 states, where these states are defined as:

STATE	PHYSICAL STATUS
1	0 generators inoperative
2	1 generator inoperative
3	2 generators inoperative
.	
.	
i	(i-1) generators out
.	
N + 1	All generators inoperative

We can now express the generating capacity available for each of these states. Let G be total installed capacity and C the capacity of each generator. (G = NC):

STATE	GENERATING CAPACITY
1	$G = NC$
2	$(N-1)C$
i	$(N+1-i)C$
N + 1	0

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In order to proceed further with this model, we must first postulate an operating condition for the ship. We will assume that the ship is operating at sea, under normal wartime cruising conditions, with no generators unavailable because of routine maintenance. For this model we exclude the possibility of battle damage, restricting the possible causes of generator outage to equipment malfunction or personnel error.

We further assume that through study of existing installations we may postulate a probability of failure for any one generator, or outage rate, where we define this probability to be:[6]

$$p = \lim_{T \to \infty} \frac{\text{Hours in T on forced outage}}{\text{Total hours in T}} \qquad \text{(IV-2)}$$

where T is the period of observation.

We will further assume that this probability is the same for all N generators and, of more importance, that failure of one or more generators will not affect the value of p for the remaining generators. The probability that a generator is operative is then

$$q = 1 - p \qquad \text{(IV-3)}$$

We can now easily calculate the state probabilities for the generating plant, that is, the probability that the plant will be in any one of the physical states that we have defined as 1 through N + 1, at any given time of observation. If we designate state probability of being in state i as π_i, we have, from the binominal distribution:[3]

$$\pi_1 = q^N$$

$$\pi_2 = \frac{N!}{(N-1)!} pq^{N-1} = Npq^{N-1}$$

$$\pi_i = \frac{N!}{(i-1)!\,(N+1-i)!} p^{i-1} q^{N+1-i} \qquad \text{(IV-4)}$$

and we see that

$$\pi_{N+1} = p^N$$

We may now compute system reliability from Equation IV-1 by considering probability of loss of load, $P_D(t)$. As we have stated previously, the probability of the occurrence of an event (in this case loss of load) is equal to the sum of the probabilities of all possible ways in which the event can occur. The system can assume any one of the discrete N+1 states. Then the probability of loss of load is equal to the sum of the products of state probability times probability of loss of load given that state, or in an equation:

$$P_D = \sum_{i=1}^{N+1} \pi_i P_i \qquad \text{(IV-5)}$$

where P_i = probability of loss of load, given that the system is in state i.

Now P_i is simply the probability that the total load demand will be equal to or greater than the capacity of the system in state i. We have found an expression for system capacity in state i to be (N + 1 - i)C. Then from the definition of the first probability density function for continuous random variables, we have

$$P_i = \int_{(N+1-i)C}^{\infty} p(L)dL \qquad \text{(IV-6)}$$

where p(L) = probability density function of total load, derived from load analysis

and finally we have for system reliability:

$$R = 1 - P_D$$

$$= 1 - \sum_{i=1}^{N+1} [\pi_i \int_{(N+1-i)C}^{\infty} p(L)dL] \qquad \text{(IV-7)}$$

The symbol for time dependence has been omitted here, but we must remember that in general π_i and $p(L)$ and hence R are functions of time.

Let us examine for a moment Equation IV-7. This is an expression for system reliability as a function of the following basic quantities:

a. Estimated total load description, $p(L)$.

b. Individual generator outage rate, or probability of failure, p.

c. Number of generating units, N.

d. Generating capacity of individual units, C.

e. The product of (c) and (d), $NC = G$, total installed generating capacity.

The equation allows us to examine the effect of variations in the above quantities, either singly or in groups, on overall system reliability. Furthermore, we have in C a parameter which gives us, for any N, a direct measure of system weight, space requirements, and cost, insofar as it is affected by the generators and associated switchgear, since we have readily available design information giving us cost, weight, and space requirements for individual generators as a function of KW rating.

Before considering the problem of using the material de-

veloped to date in a logical design procedure, we should consider the effects on the system model of variations from some of our many assumptions.

B. Consideration of Battle Damage and Emergency Generating Reserve Capacity

The present Navy design practice in regard to battle damage is to stipulate, for various assumed conditions of damage, the total capacity which must remain, based upon the initial estimate of load under battle conditions. For example, it is specified that if damage should result in the loss of generators in two adjacent compartments, the remaining generators and emergency generators shall be capable of carrying the estimated battle load.

It would be possible for an operations research team to set up a battle damage model which could predict the probability of damage to any given piece of shipboard equipment under battle conditions. But since it would be based upon the effectiveness of ships and weapons which have not actually been tested in modern warfare, one would have no measure of the validity of the results and would experience considerable doubt in using them.

What the designer must decide in the design procedure which has been proposed is:

What is a desirable lower limit on system reliability? It is meaningless, after all, to speak of 100% reliability. Such a condition does not exist in a mechanical system. We can say 100% reliability (given that all loads are not operated at peak, power simultaneously, no personnel errors are

committed, and no mechanical malfunction of generators occurs), but this is not 100% reliability, or R = 1, as we have defined it.

Assuming then that a lower constraint on reliability has been established, a reasonable extrapolation from present practice would be to require that under a given condition of damage the reliability of the remaining system, which would now include the emergency generators, should be equal to or greater than this level. We have now a problem which can be handled analytically.

Once the possibility of battle damage has been introduced, under present design philosophy we may allow, in effect, an increase in the total installed generating capacity by an amount equal to the emergency generator capacity installed. If we now postulate as battle damage the loss of some segment of this new system, we may carry out a reliability calculation on the remainder in essentially the same manner as has been done in the discussion preceding this section. We would expect that the emergency generators would have a somewhat different value for outage rate, also that their rating would not be equal to C. Such a calculation, as will be illustrated in our sample design, would lead to a choice of emergency generator capacity. This, of course, would then have to be checked against the "emergency" shipboard load estimate, to insure that this small sub-system would also possess a sufficiently high value of reliability.

A numerical example of a simplified system study has been carried out in Appendix B.

C. Preliminary Design Procedure

The procedure to be used in preliminary design would be in many ways quite similar to that presently employed. The steps which would be followed are listed below in sequential form:

1) Tabulation of information that is available concerning electrical power consuming equipment to be installed in the ship.

2) Specification of operating conditions to be considered in carrying out the load analysis. In the case of the typical surface combatant warship, this would include as a minimum a period extending from normal wartime cruising through a transition to full battle condition; a period when the ship is assumed to be operating emergency loads only; and possibly an "at anchor" condition as is presently done, though this seems of slight interest in initial generator sizing.

3) Description of component loads as stochastic power-time signals, considered either individually or in groups where possible, so that a characteristic value of mean and variance can be estimated for each of the conditions in (2) above.

4) Summation of these values to determine the total load statistical description for each of the operating conditions. Here again, these values of mean and variance must be considered as time functions, since we are interested in the duration of maximum expected load, as well as the magnitude.

5) Through application of the assumption of a normal

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

probability density function for the total load, carry out a system study as discussed on the preceding pages.

V. DISCUSSION AND RECOMMENDATIONS

This thesis has presented only in outline form the proposal for a quantitative system optimization procedure. The general method of attack, some of the theoretical aspects of the method, and some possible mathematical models for component and total load descriptions have been sketched in rather broad and general terms. Approximations of considerable importance have been made in the theoretical derivation upon whose validity depends the worth of the final results. For example, it is often found that the normal approximation to some unknown density function often becomes less valid at the tails of the normal curve. Since this is the region of the curve with which we are concerned at high levels of reliability, the importance of checking this approximation becomes obvious.

Aside from the experimental evidence which would be necessary to check these assumptions, actual application of this method depends upon the collection of data on component load behavior and generator reliability which is in many cases not presently available, at least in the form which we would require.

The following specific steps are recommended as necessary to the verification of this method and its evolution into a practical design tool:

1. Initiation of a program of data collection aimed at the investigation of the load-time behavior of components and total load or segments thereof. This data would be in

[illegible]

[illegible]

[illegible]

[illegible]

the form of continuous load - time plots obtained from a recording wattmeter or similar instrument.

2. Analysis of this data to determine a logical classification scheme for various loads and load behavior within these classifications, so that a load analysis based upon predicted behavior would be possible.

3. Analysis of existing data to determine reasonable estimates for marine generator outage rates, both turbine and diesel driven.

4. Development of a digital computer program for the load and system analysis calculations. This requires the development of a suitable model for load behavior in each sub-classification, similar to the ones that have been developed for induction motors. Obviously, application of the method would be immeasurably more economical on the basis of engineering time with such a program.

5. Investigation of the possibility of the development of a statistical wattmeter, which would record long-time mean and deviation of component load requirements, and thus simplify the process of data collection.*

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* A statistical voltmeter, designed to perform this function on system voltage has been developed recently in France.[7]

VI. APPENDIX

APPENDIX A

NUMERICAL EXAMPLES OF LOAD ANALYSIS

The author's original intention had been to apply the method to an actual ship's electrical system for illustrative purposes and possible verification of the theory. Unfortunately, quantitative data that would be necessary for a study of this sort at the present time is either extremely difficult to find or simply not available. A study could have been made, of course, by employing a series of approximations for probability density functions, and the best possible estimates of the remaining data necessary for such a study. However, aside from the fact that time was not available for the work, it was felt that such a procedure, though possibly of some value for illustrative purposes, would not actually have any real value insofar as a validation of the method was concerned. A comparison of these results with results of a design based on present practice would indicate only a relative difference between two approximations.

The comparison that is needed for these two methods is one which would have as a reference the actual electrical load structure as it exists on a completed ship, or actually series of ships. Only in this fashion can we determine the relative merits of competing design methods. Since an illustration of the application of the method to some typical loads is obviously needed, the decision was made to consider in some detail the procedure for analysis of a component load

from each of the classes discussed in Chapter 3.

Example 1: Induction Motor - Immediate Effect

Consider a typical missile launcher drive motor, with the following specifications:

Induction Motor (Squirrel cage)

40 HP

440 v., 3 phase, 60 cycle

If we assume an efficiency for the motor of 0.80, then the rated kilowatt power requirement is:

KW input = 40 hp x 746 watts/hp x 1 KW/1000 watts x 1/0.8

= 37.3 KW

Figure X represents a plot of per unit power vs. per unit speed for a typical induction motor[8]. We see that the starting transient power requirement is 3 per unit rated, or in this case 112 KW. If we assume for this application that the steady state power requirement of the motor is 0.2 per unit, then for N/n in Eq. III-18 we have 15.0.

Let the starting transient time for this motor be 0.2 min. Then with the starting time probability density function a Gamma Distribution, with $C_1 = 4.0$, $C_2 = 2.0$, from Equations III-18 through III-22 we can obtain values of E(k) and σ_k versus time over the range of interest, approximately 3 minutes. The results of this series of calculations are shown in graphical form in Fig. XI . Note that as we would expect, the mean value of load approaches steady state value, n, and the deviation approaches 0 for increasing time.

The peak value of both E(k) and deviation occur at time 0.7 minutes after the initiation of the operating condition

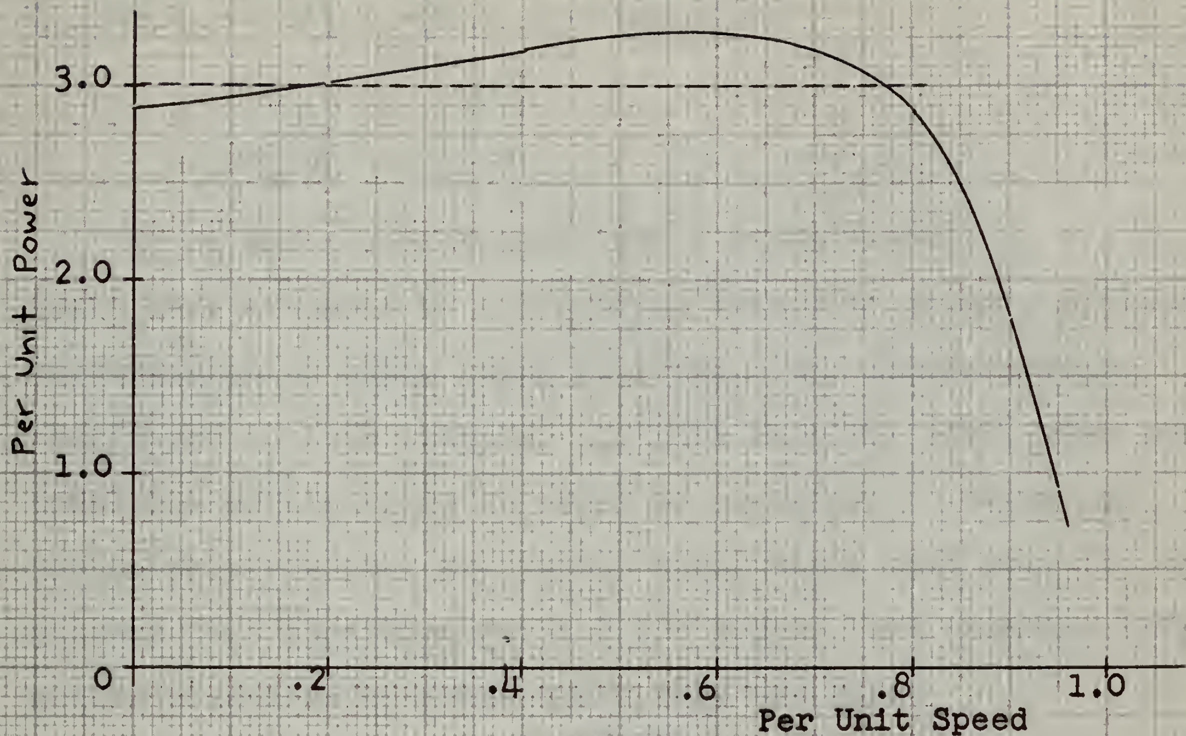

FIGURE X
Induction Motor Speed-Power Curve

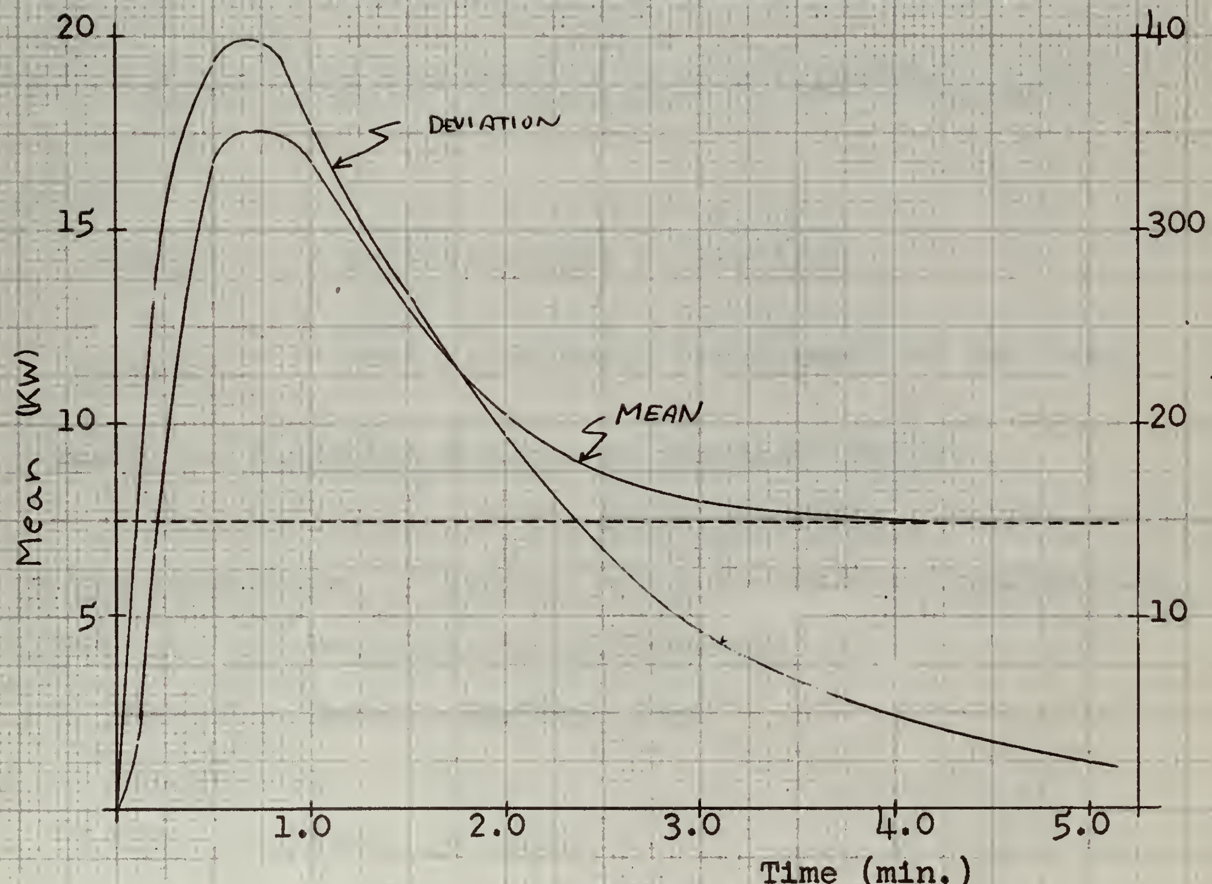

FIGURE XI
Sample Mean and Deviation vs.
Time Plot

transition. For this time t and for the assumptions for T, C_1 and C_2 as noted above, we find that

$$E(k)/n = 2.405$$
$$\sigma_k^2/n^2 = 26.67$$

It is quite conceivable that a given electrical system would have a number of these drive motors of roughly similar characteristics which would fall under the classification considered in this example. If this were the case, with the constants we have used it would be necessary to carry out the calculation only at t = 0.7 min., since this would be the maximum load condition for all the motors, and then the total statistical load estimate would have the form:

$$E(L) = \sum_i E(k_i) = \sum_i 0.481(K_{rated})_i = 0.481 \sum_i (K_{rated})_i$$

$$\sigma_L^2 = \sum_i \sigma_{K_i}^2 = \sum_i [1.065(K_{rated})_i]^2 = 1.135 \sum_i (K_{rated})_i^2$$

where

$E(K_i)$ = expected value of ith load

$(K_{rated})_i$ = name plate power requirement of ith load

Example 2: <u>Induction Motor - No Immediate Effect</u>

Consider the case of the drive motor for a ship's service LP air compressor. A typical motor for such an application might have the following specifications:

Induction Motor - Squirrel cage

Rated Power - 15 HP

440 v, 3 phase, 60 cycle

Assuming an efficiency of 0.8, full rated power requirement for this motor would be

$$15 \times 746/1000 \times 1/0.8 = 14.0 \text{ KW}$$

Studies (Technical Report, BNSY, USS TURNER) show that the LP air compressor, an intermittent duty device, may be started on the average of 210 times per day. Furthermore, the average charging time in any given cycle is approximately 3 min. Then from Eq. III-26, we have:

$$P_t = \frac{210 \times 3}{24 \times 60} = 0.438$$

Hence

$$P_a = 1 - P_t = 0.562$$

In this particular application, we will assume that the motor will be running at rated power output during the entire charging cycle. Then in Eq. III-24 and Eq. III-29 we have for E(N) and $E(N^2)$ 14.0 and 196 respectively. Then from Eq. III-22,

$$E(k)\ \frac{1}{2}\ (0.562)(0) + (0.438)(14) = \underline{6.13 \text{ KW}}$$

And from Eq. III-29, we have

$$\sigma_k^2 = (0.438)(196) - 2(0.562)(0.438)(0)(14) - (6.13)^2$$

$$\sigma_k^2 = 48.4 \text{ KW}^2$$

The contribution of this load to the overall load analysis, then, is a mean of 6.1 KW and variance of 48.4 KW^2. Note that these values are not functions of time, since we have considered a steady state load.

Other motor loads which would have the same general behavior are the air conditioning and refrigeration compressor

drive motors and the steering gear pump motors. The compressor drives are almost exactly similar to the air compressor drive, since they operate on an on-off type of cycle, varying between either full rated load or off as regards power requirement. The steering engine pump motor is somewhat different, in that the motor is continuously running and hence has at all times a finite value of load requirement. Another important distinction is the fact that the motor will exhibit a range of power requirements while it is responding to a rudder positioning order, depending on the magnitude of the order and speed with which it is applied to the system. Obviously, a number of ordnance loads, in their steady state behavior, are essentially similar to the steering gear pump motor.

APPENDIX B

NUMERICAL EXAMPLE OF SYSTEM STUDY

The following example will consider a hypothetic preliminary design for what might be a DL type ship, with a total rated installed electrical load of 5000-6000 KW. Numerical data is supplied purely for purposes of illustration and does not reflect a detailed study of any particular ship.

The study will begin with the assumption that the load analysis has been completed with the results as shown in Fig. XII. The figure is a plot of mean and deviation of total load vs. time for the period from normal cruising through General Quarters to battle condition. The effect of projected load growth has been considered by an increase in the values determined for mean by a factor of 1.2, as is present practice. It is assumed that the standard deviation will be unaffected by load growth. This problem has not been considered in any detail, however. One argument that might be used in favor of this assumption(an argument that can only be tested by actual study of completed systems) is as follows: The numerical examples considered indicate that the major contribution to the value of total load variance may come primarily from motor loads (machinery and weapons drive). Experience indicates that the major load growth in electrical systems has taken place in electronics equipment, air conditioning, and fire control. As the examples in Appendix A illustrate, these loads present a rather constant load re-

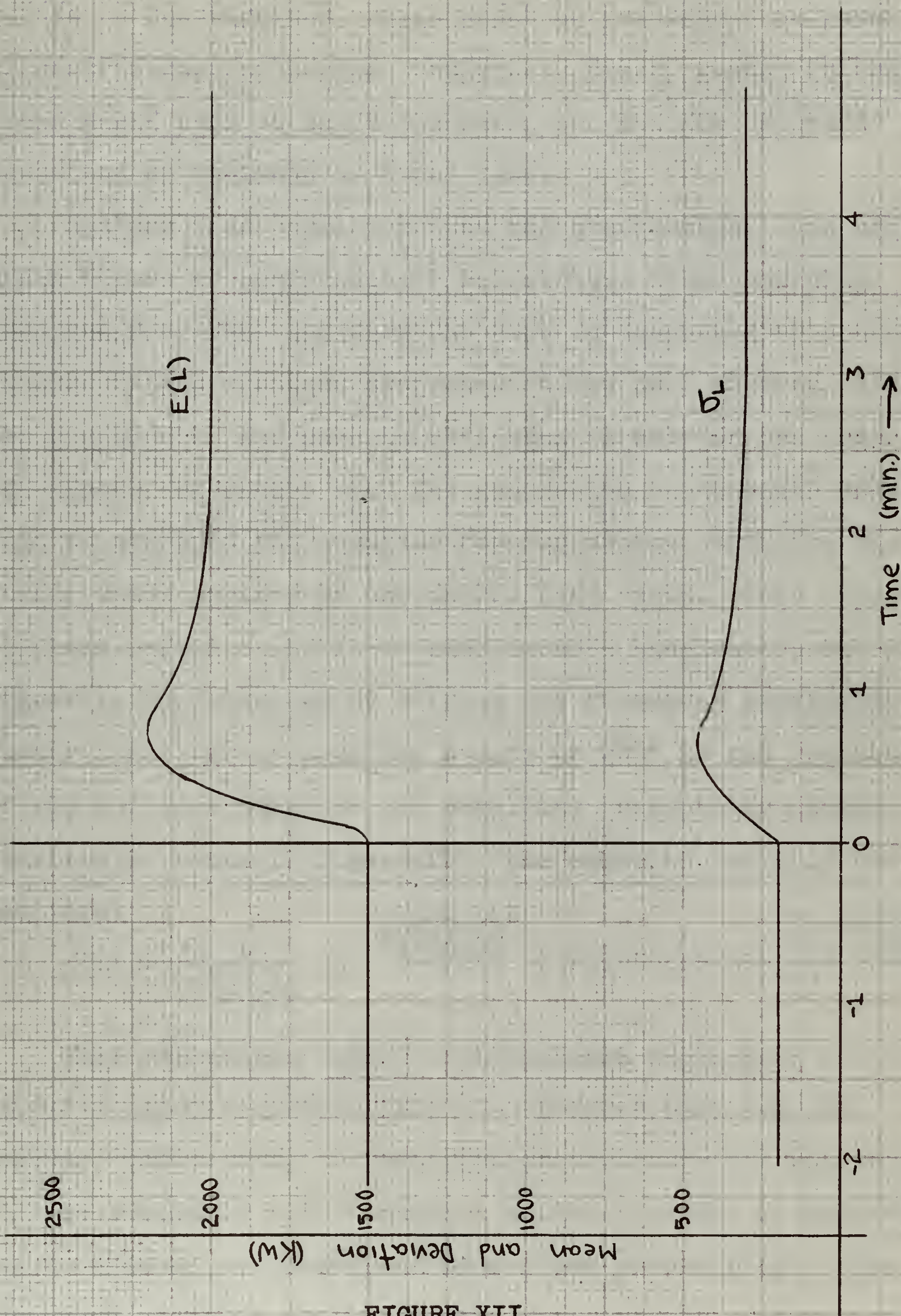

FIGURE XII

Assumed Statistical Description of
Total Load Behavior

quirement, relative to motor loads, and hence contribute little to the variance, which might be considered an index of uncertainty in the load estimate. Hence, growth in these areas would tend to shift the mean, but not the standard deviation or variance of total load.

Maximum load condition from the hypothetical load analysis occurs at approximately 0.7 minutes after sounding "General Quarters" and from the plot is described by a mean of 2200 KW and standard deviation of 450 KW. However, since the duration of the peak in this case is only of the order of one minute, we assume that the generating plant could handle such an overload and consider for our design condition the steady state portion of the battle load curve. This transient overload period is still of considerable interest to the designer in the problems of voltage and frequency regulation. Steady state battle load has a mean of 2000 KW and deviation of 300 KW. The shape of the resulting probability density function is shown in Figure XIII. The equation for p(L) has the form:

$$p(L) = \frac{1}{\sqrt{2\pi}\,300} e^{-\frac{(L-2000)^2}{2(300)^2}} = 13.33 \times 10^{-4}\, e^{-\frac{(L-2000)^2}{1.8 \times 10^5}}$$

This expression cannot be integrated explicitly to obtain, for example, probability of L greater than 3000 KW. However, tabulations of these integrations for the unit normal are available and, through a suitable change of variable, may be entered to determine the desired probability. This change of variable has the form

$$u = [L - E(L)]/\sigma_L$$

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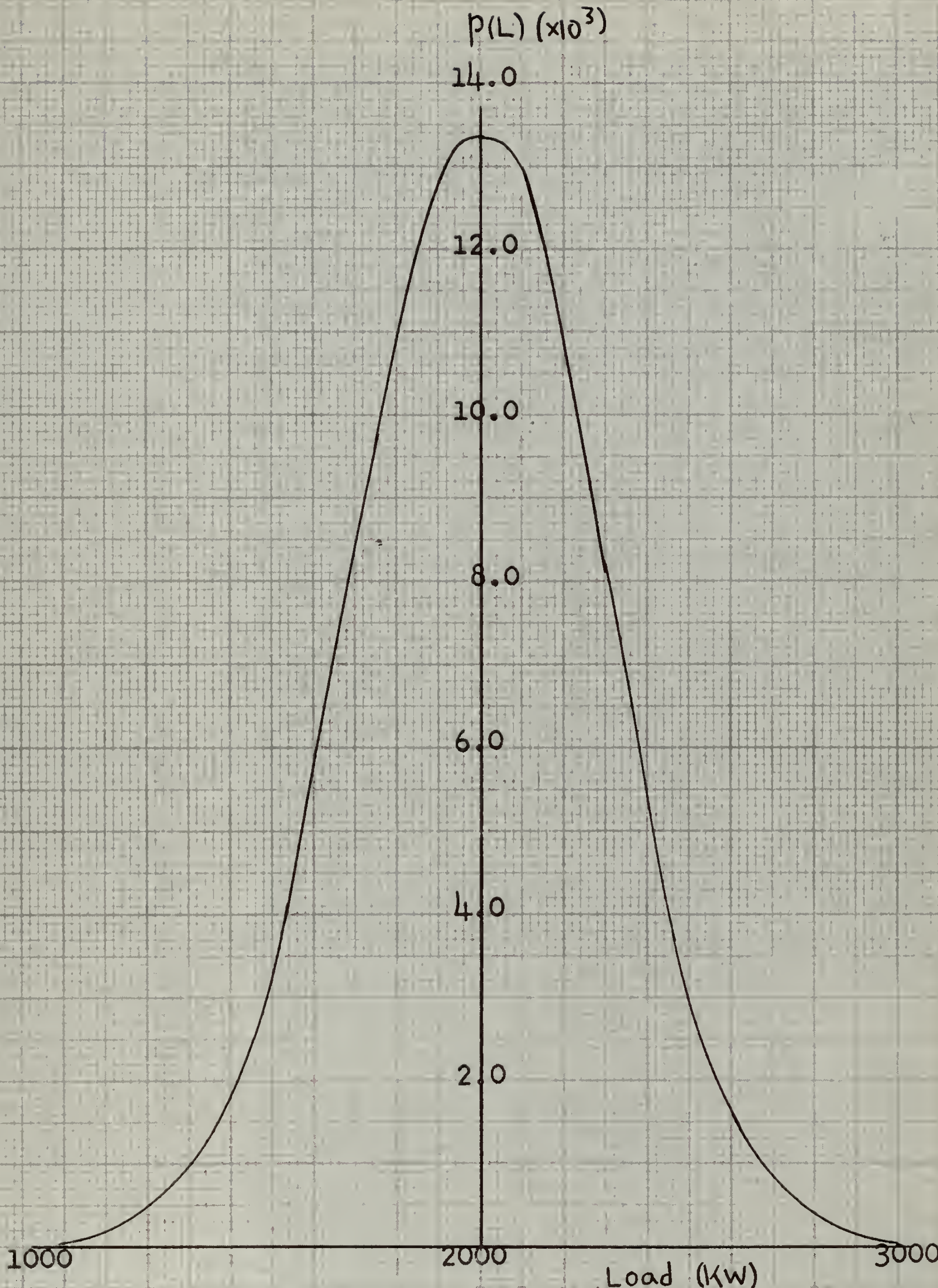
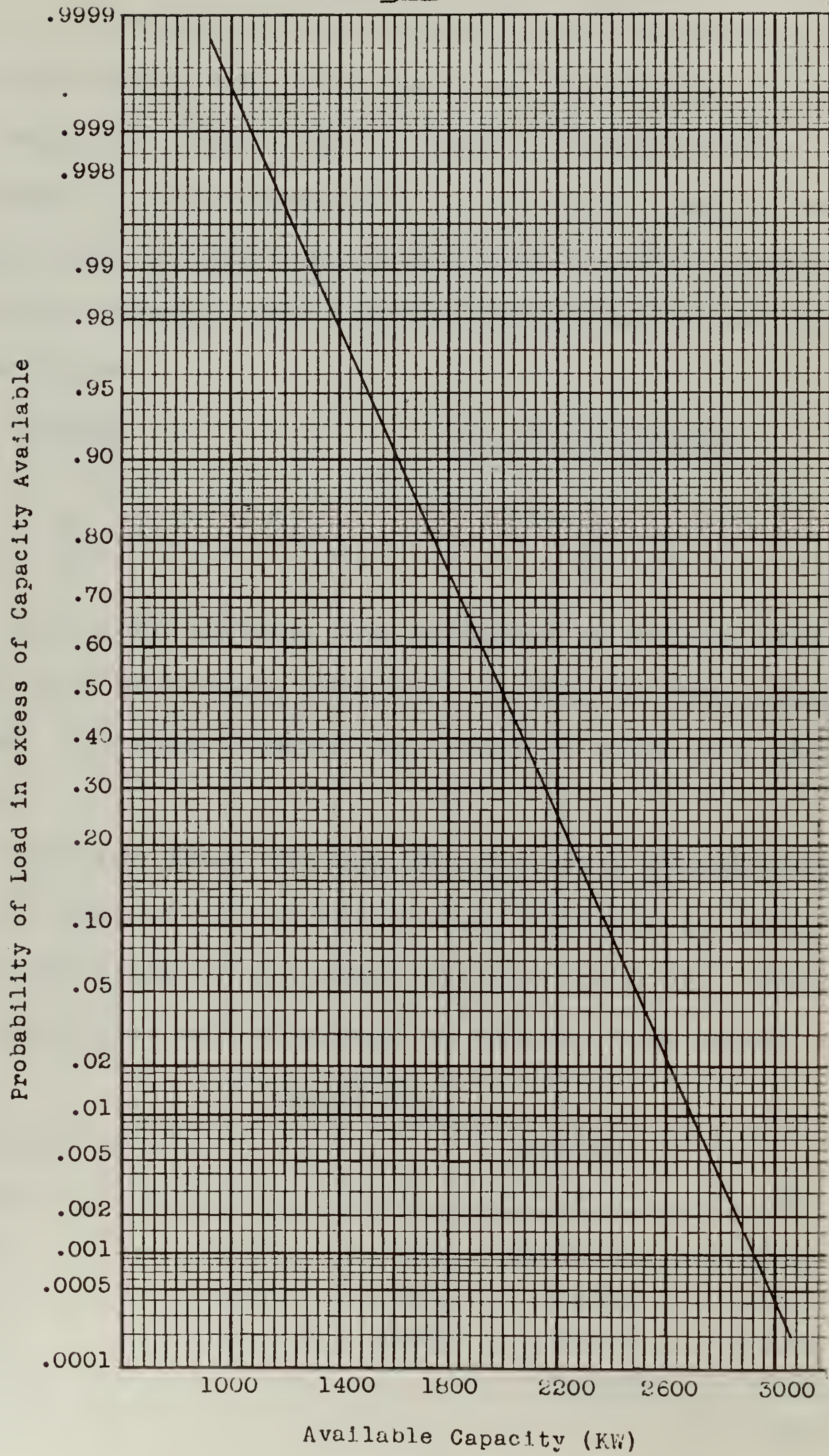


FIGURE XIII

Plot of Normal Density Function
for = 2000, = 300

Figure XIV


.9999
.999
.998
.99
.98
.95
.90
.80
.70
.60
.50
.40
.30
.20
.10
.05
.02
.01
.005
.002
.001
.0005
.0001
Probability of Load in excess of Capacity Available
1000
1400
1800
2200
2600
3000
Available Capacity (KW)

A rough plot of the distribution function or probability that the total load exceeds some value L is shown in Figure XIV as P(L) vs. L. These are the probabilities that must be used in the system design procedure that was outlined in Section IV.

Excluding the possibility of battle damage, that is, investigating only the capability of the intact ship's service generating plant to carry the battle load, we will consider in this simple example four possible solutions to the design problem as below:

SYSTEM	NO. OF GENERATORS
A	1
B	2
D	3
E	4

For these four systems, the variation of reliability with individual generating capacity, C, will be determined and plotted, and these results will then be used in a system comparison.

From Equation IV-4, Section IV, the state probabilities for the four systems can be calculated, assuming an outage rate, or probability of generator failure, of 0.02. These state probabilities are presented in tabular form below:

TABLE B-1

STATE PROB., π	SYSTEM			
i	A	B	D	E
1	0.98	0.9604	0.9412	0.9224
2	0.02	.0384	.05762	.0753
3		.0004	.00118	.0023
4			.000008	.0000314
5				.00000016

Now from Equation IV-7 system reliability for each of the above systems can be calculated as a function of C, remembering to enter the unit normal tables with the variable

$$x = [(N + 1 - i)C - E(L)]/\sigma_L$$

The range of C investigated has been limited so that minimum C is equal to E(L)/N.

A sample calculations, for Systems A and B, are tabulated in Tables B-2 and B-3. The calculations themselves are simply a straightforward application of the equations which were developed in Section IV.

The results for this particular set of calculations are shown in graphical form in Figure XV . Here the reliability for each system is plotted as a function of total installed generator capacity.

Discussion of Figure XV:

Assuming that we are concerned with the problem of sizing a generating plant to carry the battle load we have postulated for this example, considering only the possibility of generator outage due to equipment malfunction or personnel error, this plot would be of considerable assistance in making a decision between the four systems under consideration.

The initial design decision required would be the establishment of a desirable level of reliability. This design reliability, plotted as a horizontal line on the Figure, would intersect each of the reliability vs. capacity curves of the four systems at some value of total capacity.* Knowing the

*Note that the reliability of System A is limited to 0.98 since assumption of a generator outage rate of .02 leaves this system, with one generator always this prob. of loss of load.

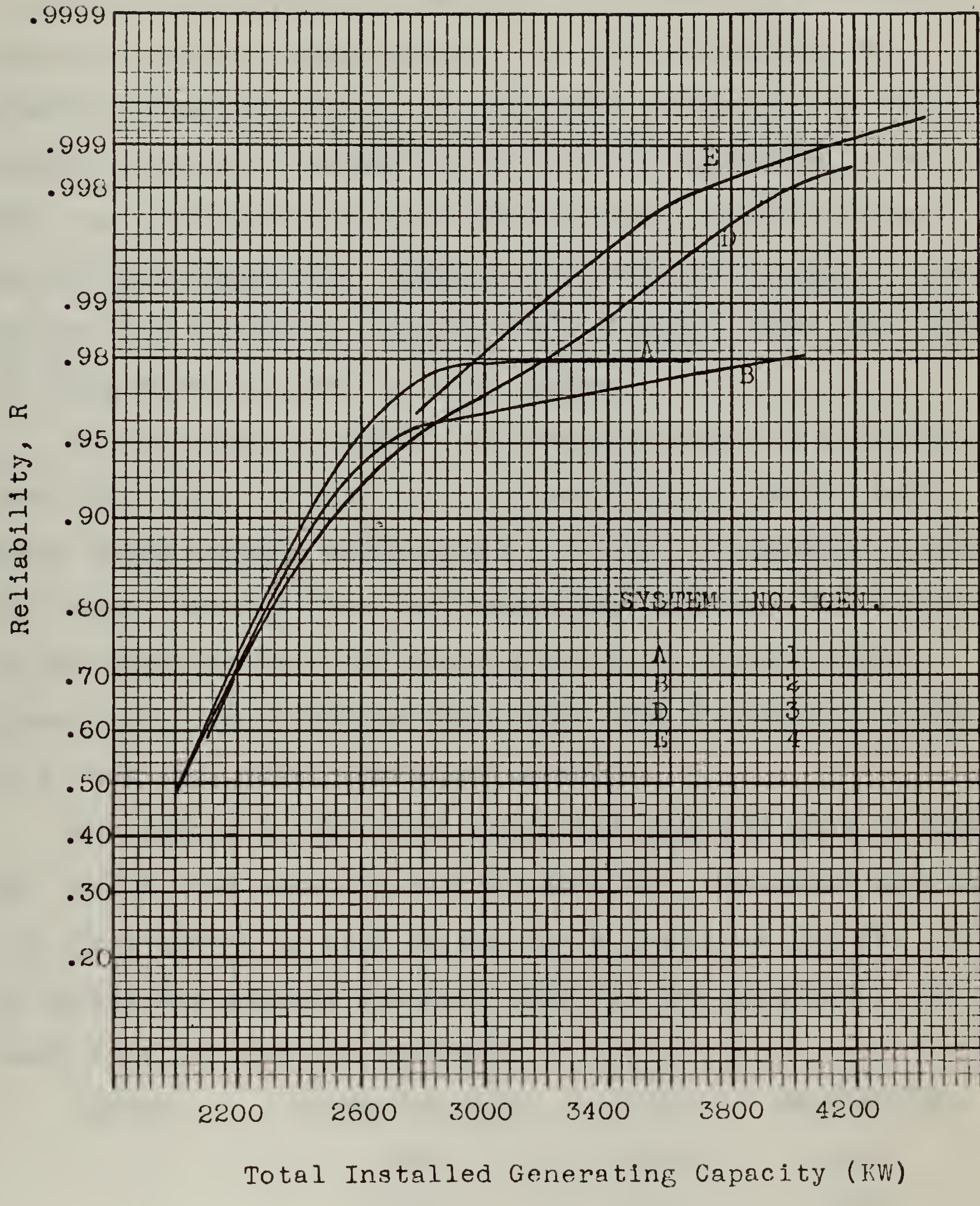


Figure XV

Reliability vs. Installed Generator Capacity
System Example

total capacity required of the systems, and the number of generators per system, we can then calculate the required generator size for each system. If we were dealing with standard size generators, as is present practice, we might choose the nearest standard size to this calculated value, with due consideration given to the effect of this choice on system reliability. Knowing the required generator size, we are in a position to estimate the required cost, and weight of the generating plant of each system.

As an example, suppose that we have chosen a reliability level of 0.998. This gives a probability of loss of load under these loading conditions of .002 or 12 minutes in every 100 hours. We see from the figure that system A and B are out of the question with our assumed generator outage rates, since A will never attain this level, and B reaches it only at a very high total installed capacity.

For system D, the total installed capacity must be 4000 KW. For E, the capacity must be 3660 KW. Generator size for D is then 4000/3 or 1330 KW and for E 3660/4 or 915 KW. If we picked the nearest standard size for the generators, we would find for the two systems:

SYSTEM	GENERATOR SIZE	TOTAL	RELIABILITY
D	1500	4500	.999
E	1000	4000	.9988

On the basis of the somewhat sketchy cost and weight data available to the author, it was found that the cost for a typical 1000 KW marine turbo-generator might be 180,000 dollars (450V/60 cycle). The data yielded the same cost

figure for the 1500 KW generator. Tabulated below, we have for the system comparison:

		UNIT			TOTAL	
SYSTEM	RATING	COST	WEIGHT	RATING	COST	WEIGHT
D	1500	180,000	28,000	4500	540,000	84,000
E	1000	180,000	26,000	4000	720,000	104,000

On the basis of the simplified study we have made, since we have applied a consistent measure of system performance (reliability) to both systems and found them approximately equal, the logical choice between the two systems is System D.

TABLE B-2

Reliability Calculations for System A

C	State	G	P_i	π_i	$\pi_i P_i$	$\Sigma \pi_i P_i$	R
2000	1	2000	.5	.98	.490	.51	.490
	2	0	1.0	.02	.02		
2200	1	2200	.254	.98	.2489	.2689	.7311
	2	0	1.0	.02	.02		
2400	1	2400	.0918	.98	.08996	.10996	.8900
	2	0	1.0	.02	.02		
2600	1	2600	.0228	.98	.02234	.0423	.9577
	2	0	1.0	.02	.02		
2800	1	2800	.0039	.98	.00382	.0238	.9762
	2	0	1.0	.02	.02		
3000	1	3000	.0004	.98	.00039	.02039	.9796
	2	0	1.0	.02	.02		

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		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

TABLE B-3

Reliability Calculations for System B

C	State	C	P_i	π_i	$P_i\pi_i$	$P_i\pi_i$	R
	1	2000	.5	.9604	.4802		
1000	2	1000	.9996	.0384	.03838	.5186	.4814
	3	0	1.0	.0004			
	1	2200	.254	.9604	.2439		
1100	2	1100	.9987	.0384	.03835	.28265	.71735
	3	0	1.0	.0004	.0004		
	1	2400	.0918	.9604	.08816		
1200	2	1200	.9962	.0384	.03825	.12681	.8732
	3	0	1.0	.0004	.0004		
	1	2600	.0228	.9604	.0219		
1300	2	1300	.9901	.0384	.03802	.06031	.93969
	3	0	1.0	.0004	.0004		
	1	2800	.0038	.9604	.00365		
1400	2	1400	.9772	.0384	.03752	.04157	.95843
	3	0	1.0	.0004	.0004		
	1	3000	.0004	.9604	.00038		
1500	2	1500	.9525	.0384	.03658	.03736	.96264
	3	0	1.0	.0004	.0004		
	1	4000	0	.9604	0		
2000	2	2000	.5	.0384	.0192	.0196	.9804
	3	0	1.0	.0004	.0004		

[illegible]

APPENDIX C

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